

DEPARTMENT OF PRE-UNIVERSITY EDUCATION

MODEL QUESTION PAPER FOR ANNUAL EXAMINATION APRIL-2022

II PUC

SUB: MATHEMATICS (35)

TIME: 3 Hours 15 Minutes MAX. MARKS: 100

Instructions :

- (i) The question paper has five parts namely A, B, C, D and E. Answer all the parts.
- (ii) Use the graph sheet for the question on Linear programming in PART E.

PART – A

Answer any TEN questions 10 X1=10

1. Give an example of a relation which is symmetric and transitive but not reflexive.
2. Define a binary operation.
3. Find the principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$.
4. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x.
5. Define a row matrix.
6. Find the value of x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$.
7. If $y = e^{\cos x}$, find $\frac{dy}{dx}$.
8. If $y = \sin(x^2 + 5)$, find $\frac{dy}{dx}$.
9. Find $\int (2x^2 + e^x) dx$.
10. Evaluate $\int_2^3 \frac{1}{x} dx$.
11. Find the unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

12. Write two different vectors having same magnitude.
13. Write the direction cosines of x-axis.
14. Define feasible region of a linear programming problem.
15. Find $P(A|B)$, if $P(B) = 0.5$ and $P(A \cap B) = 0.32$.

PART- B

Answer any TEN questions 10 X2=20

16. Show that the signum function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$

is neither one-one nor onto.

17. Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$.

18. Write the domain and range of $y = \tan^{-1} x$.

19. Find the values of x, y and z from the equation $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$.

20. Find equation of line joining (1, 2) and (3, 6) using determinants.

21. If $x^2 + xy + y^2 = 100$, find $\frac{dy}{dx}$.

22. If $x = at^2$, $y = 2at$, find $\frac{dy}{dx}$.

23. Differentiate $\sin(\cos(x^2))$ with respect to x.

24. Find the slope of tangent to curve $y = x^3 - x + 1$ at the point whose x-co-ordinate is 2.

25. Find $\int \frac{(\log x)^2}{x} dx$.

26. Find $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$.

27. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 x dx$.

28. Find the order and degree of the differential equation $y^1 + y = e^x$.

29. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

30. Find the area of the parallelogram whose adjacent sides are given by the vectors

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}.$$

31. Find the intercepts cut-off by the plane $2x + y - z = 5$.

32. Find the distance of the point $(-6, 0, 0)$ from the plane $2x - 3y + 6z - 2 = 0$

33. The random variable X has a probability distribution $P(X)$ of the following form where k is some number. Find the value of k .

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

PART - C

Answer any TEN questions 10 X 3 = 30

34. Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$

is reflexive but neither symmetric nor transitive.

35. Prove that $2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$.

36. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ using elementary operations.

37. Verify that the value of the determinant remains unchanged if its rows and columns are

interchanged by considering third order determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

38. If $xy = e^{x-y}$ find $\frac{dy}{dx}$.

39. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ find $\frac{dy}{dx}$.

40. Verify mean value theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$

where $a = 1$ and $b = 4$.

41. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) increasing (b) decreasing.

42. Find $\int \frac{xe^x}{(1+x)^2} dx$.

43. Evaluate $\int \frac{1}{(x+1)(x+2)} dx$.

44. Evaluate $\int_0^2 e^x dx$ as the limit of a sum.

45. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

46. Form the differential equation representing the family of curves $y = a \sin(x+b)$, where a, b are arbitrary constants.

47. Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.

48. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

49. Find x such that the four points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are co-planar.

50. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

51. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART -D

Answer any SIX questions

6X5=30

52. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined $f(x) = \frac{x-2}{x-3}$.

Is f one-one and onto? Justify your answer.

53. Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$.

Show that $f: \mathbb{N} \rightarrow S$ where, S is the range of f is invertible. Find the inverse of f .

54. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)^T = A^T + B^T$ (ii) $(A-B)^T = A^T - B^T$

55. Solve system of linear equation, using matrix method.

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

56. If $y = 3 \cos(\log x) + 4 \sin(\log x)$ Show that $x^2 y_2 + x y_1 + y = 0$.

57. The length x of a rectangle is decreasing at the rate of 5cm/minute and the width y is increasing at the rate of 4cm/minute. When $x = 8$ cm and $y = 6$ cm find the rates of change of

a) the perimeter

b) the area of the rectangle.

58. Find the integral of $\frac{1}{x^2 + a^2}$ with respect to x and hence evaluate $\int \frac{3x^2}{x^6 + 1} dx$.

59. Find the area enclosed by the circle $x^2 + y^2 = a^2$ using integration.

60. Find the general solution of the differential equation $x \frac{dy}{dx} + 2y = x^2$ ($x \neq 0$).

61. Derive the equation of a plane perpendicular to a given vector and passing through a given point both in vector and Cartesian form.

62. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

a) at least once

b) exactly once

63. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem

PART – E

Answer any ONE question1 X10=10

64. (a) Maximise $Z = 3x + 2y$ subject to the constraints $x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$

b) If the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$, when I is 2×2

identify matrix A and O is 2×2 zero matrix. Using this equation find A^{-1} .

65. (a) Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ and hence evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$.

(b) Find the value of K so that the function $f(x) = \begin{cases} Kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases}$

is continuous at $x = 5$.

66. (a) Prove that the volume of the largest cone that can be inscribed in a sphere of

radius R is $\frac{8}{27}$ of the volume of the sphere.

b) By using properties of determinants

Show that
$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3.$$
